

Point form relativistic quantum mechanics, antiparticles and exchange currents.

L. Ya. Glozman

*The Racah Institute of Physics, The Hebrew University
of Jerusalem, Israel and Dipartimento di Fisica
Nucleare e Teorica, Universita di Pavia, Italy*¹

Abstract

It is shown that the dynamical observables calculated with the point form relativistic quantum mechanics incorporate effects of particle-antiparticle creation from the vacuum by interactions. The electromagnetic observables obtained with the point form impulse approximation include contributions from the “pair” exchange currents that are associated with the interactions between particles. This implies that the recently calculated nucleon electromagnetic formfactors with the chiral constituent quark model automatically take into account effects of “pair” exchange currents that are associated with the Goldstone boson exchange between the constituent quarks as well as with the confining interaction.

Among the various forms of relativistic quantum dynamics, introduced by Dirac [1, 2], the point form has some obvious advantages. It contains interactions in 4-momentum operators, while the six generators of Lorentz transformations (i.e. boosts and spatial rotations) are pure kinematic. This feature is rather important and allows one to perform manifestly covariant calculations of different dynamical observables once one possesses a few particle wave function in the rest frame [3]. Very recently the latter formalism has been applied to nucleon electromagnetic formfactors [4]. In that work, use has been made of nucleon wave functions obtained within the chiral constituent quark model of ref. [5] which relies on the idea of Goldstone boson exchange between the constituent quarks in baryons in the low-energy regime of QCD [6, 7]. This type of interactions between the quasiparticles (constituent quarks) allows to a simultaneous description of the low-energy baryon mass spectrum in all u,d,s flavor sectors (for a recent overview see ref. [8]). The calculation in ref. [4] is a straightforward one (following all the prescription of ref. [3]) and does not involve any adjustable parameters. All the predictions are found to be in remarkable agreement with existing experimental data. For example, the electric proton

¹ e-mail: leonid.glozman@kfunigraz.ac.at

and neutron formfactors as well as radii fit experiment within the error bars. Only magnetic moments and magnetic formfactors deviate from the experimental data by a few percent in magnitude, though the magnetic formfactors follow the required dipole form below 1 GeV. Given this success it is rather important to understand a physical content of dynamical observables calculated within the present approach.

Within this approach, which is called point form relativistic impulse approximation, at face value only particle degrees of freedom are involved and the process associated with the creation of pairs from the vacuum as well as related exchange currents are not included [3, 4]. This impression is motivated by the fact that the few particle (baryon) wave function, that should be obtained in the baryon rest frame and can be used to calculate any observable, contains only particle degrees of freedom and no antiparticle degrees of freedom are introduced. The purpose of this letter is rather limited, we are not going to calculate anything, but rather aim to give a proper physical interpretation of the results obtained in such an approach. We demonstrate that the dynamical observables like baryon electromagnetic formfactors, calculated in the manifestly covariant point form approach, do include creation of the antiparticles from the vacuum (quark-antiquark pairs) and associated exchange two- and three-body currents. In order to see it we will heavily rely on Feynman interpretation of quantum mechanics. This is in contrast to the operator formalism that is used in ref. [3].

We will begin with the brief overview of the point form formalism. The space- and time-evolution equation in the point form relativistic quantum mechanics is prescribed by the covariant equation

$$P^\mu \langle x | \Psi \rangle = i\hbar \partial^\mu \langle x | \Psi \rangle, \quad (1)$$

where P^μ is the 4-momentum operator and Ψ is an element of the Hilbert space. The 4-momentum operators satisfy the standard Poincare algebra. In this approach all four components of the 4-momentum contain interaction, which is in contrast to e.g. the instant form of relativistic quantum dynamics (or nonrelativistic Schrödinger equation) that include interaction in the Hamiltonian only. In the system rest frame the eq. (1) becomes the Scrödinger type equation that describes an evolution of the system in proper time. The spectrum of the mass operator, $M = \sqrt{P_\mu P^\mu}$, specifies the bound as well as scattering states. The n-particle basis states (i.e. n-particle system without interactions) are introduced as tensor products of single-particle states $|p_1 j_1 \sigma_1\rangle \times \dots \times |p_n j_n \sigma_n\rangle$, where p_i, j_i, σ_i are 4-momentum, spin and spin-projection of the i-th particle. Under Lorentz transformation Λ each single-particle state is transformed in a standard way

$$U(\Lambda) |p_i j_i \sigma_i\rangle = \sum_{\sigma'_i} D_{\sigma'_i \sigma_i}^{j_i} (R_W[p_i, \Lambda]) |\Lambda p_i j_i \sigma'_i\rangle, \quad (2)$$

where $R_W = R_W(p, \Lambda)$ is the corresponding Wigner rotation and $U(\Lambda)$ forms an infinite-dimensional unitary representation of the Lorentz group. It is very convenient to introduce the so-called velocity states [11]

$$|v, \vec{k}_i, \mu_i\rangle = \sum_{\sigma_i} |p_1 j_1 \sigma_1\rangle \dots |p_n j_n \sigma_n\rangle \prod_{i=1}^n D_{\sigma_i \mu_i}^{j_i}(R_W[k_i, B(v)]), \quad (3)$$

that ensure that under Lorentz transformation Λ , the system overall 4-velocity v_μ ($v_\mu v^\mu = 1$) goes to Λv , while all internal momenta \vec{k}_i (satisfying $\sum_i \vec{k}_i = 0$) and all individual spins of particles are all rotated by the same Wigner rotation. In the expression above $B(v)$ is a boost carrying p_i to $k_i = B^{-1}(v)p_i$ with $\sum_i \vec{k}_i = 0$. These states are very useful as they form the basis where all the individual spins and orbital angular momenta can be coupled to the total spin exactly as it is done in nonrelativistic quantum mechanics.

The action of the various operators on velocity states in the system that contains no interaction between particles is specified as

$$M_{fr}|v, \vec{k}_i, \mu_i\rangle = \sum_{i=1}^n \sqrt{(m_i^2 + \vec{k}_i^2)} |v, \vec{k}_i, \mu_i\rangle, \quad (4)$$

$$V^\mu |v, \vec{k}_i, \mu_i\rangle = v^\mu |v, \vec{k}_i, \mu_i\rangle, \quad (5)$$

where the 4-velocity operator V^μ is defined as $P_{fr}^\mu = M_{fr}V^\mu$. The interaction between particles in the system is introduced according to Bakamjian-Thomas construction [9] so that this interaction does not perturb the 4-velocity of the system but does perturb the 4-momentum and mass operators

$$P^\mu = MV^\mu, \quad (6)$$

where $M = M_{fr} + M_{int}$ is the sum of the free and interacting mass operator which should satisfy $[V^\mu, M] = 0$, $U(\Lambda)MU^{-1}(\Lambda) = M$, in order to provide the Poincare algebra of 4-momentum operators. In the rest frame, the 4-velocity of the system is $v = (1, 0, 0, 0)$ and the equation (1) coincides with the proper-time-dependent Schrödinger equation with the only departure from the nonrelativistic Schrödiger equation being that the nonrelativistic kinetic energy for each particle in the system should be substituted by the relativistic one. Once the proper-time dependence is extracted from the equation and the wave function, one obtains the stationary equation

$$(M_{fr} + M_{int})|\Psi\rangle = \mathcal{M}|\Psi\rangle, \quad (7)$$

with the eigenvalue \mathcal{M} and the eigenfunction $|\Psi\rangle$. It is the latter equation that is solved variationally for the 3Q system in ref. [5]. Obviously this equation as well as its solution contains only particles and there are no antiparticles. The proper time evolution of the stationary solutions is taken into account by the factor $\exp(-iM\tau/\hbar)$. Such a solution combines the space - proper-time evolution of all particles of the system in the system rest frame. It contains only positive energy solutions that propagate forward in proper time.

In order to visualize the physical content of observables obtained in the point form relativistic quantum mechanics in the following we will rely on Feynman space-time interpretation of quantum mechanics. Since in the rest frame the evolution equation (1) coincides with the Schrödinger type equation, all the physical information, including the bound state spectrum and the wave functions can be obtained from the path integral which represents a tool to calculate the time-dependent Green function of the system. Since in the following we consider the whole system to be at rest the time which is actually used, τ , is a proper time for the whole interacting system, to be distinguished from the time t in the moving frame.

The time-dependent Green function of the individual particle in the system, that describes propagation of the particle from the space-time point x to the space time point x' , can be obtained from the integral equation

$$G(x', x) = G_0(x', x) + \int d^4x'' G_0(x', x'') V(x'') G(x'', x), \quad (8)$$

where the interaction of our particle with the other particles of the system at the space-time point x'' is given by $V(x'')$ and the free Green function $G_0(x' - x)$

$$G_0(x' - x) = -i \int \frac{d\vec{k}_i}{(2\pi)^3 2E_k} \exp(i\vec{k}_i(\vec{x}' - \vec{x})) \exp(-iE_k(\tau' - \tau)) \Theta(\tau' - \tau), \quad (9)$$

where $E_k = \sqrt{\vec{k}_i^2 + m_i^2}$.

From now on we shall follow a very elegant lecture “The reason for antiparticles”, given by Feynman [10] which is devoted to Dirac. He proves there the following: “If we insist that particles can only have positive energies, then you cannot avoid propagation outside the light cone. If we look at such propagation from a different frame, the particle is traveling backwards in time: it is an antiparticle. One man’s virtual particle is another man’s virtual antiparticle.”

Consider for simplicity the amplitude $G(x', x)$ that is quadratic in interaction $V(x)$. Such an amplitude contains a free propagation of the particle to the point x_1 , interaction $V(x_1)$ with the other particles of the system at this point, then a free propagation of the particle from this point to the point x_2 , interaction $V(x_2)$ at that point and a free propagation afterwards. This amplitude involves an integration over all possible free trajectories that connect points x_1 and x_2 , that is an integration over 3-momentum \vec{k}_i of the *virtual* particle. Then we have to integrate over all possible x_1 and x_2 provided that $\tau_2 > \tau_1$:

$$\int d^4x_1 d^4x_2 \Theta(\tau_2 - \tau_1) \int \frac{d\vec{k}_i}{(2\pi)^3 2E_k} \exp(i\vec{k}_i(\vec{x}_2 - \vec{x}_1)) \exp(-iE_k(\tau_2 - \tau_1)) a(x_1)b^*(x_2), \quad (10)$$

where the functions $a(x_1)$ and $b^*(x_2)$ contain all the information about the interaction at the points x_1 and x_2 respectively,

$$a(x_1) = V(x_1)\phi_0(x_1)\sqrt{2E_K}, \quad (11)$$

$$b(x_2) = V(x_2)\phi_0(x_2)\sqrt{2E_K}, \quad (12)$$

and $\phi_0(x_1)$ and $\phi_0(x_2)$ are initial and final plane waves.

The integration over \vec{k}_i can be turned into the integration over $\omega = E_k$ and defining $F(\omega) = 0$ for any $\omega < m_i$, the integral over \vec{k}_i in (10) is reduced to the following integral:

$$f(\tau) = \int_0^\infty e^{-\omega\tau} F(\omega) d\omega, \quad (13)$$

where the dependence on x_1 and x_2 is absorbed into $F(\omega)$. Feynman makes use of the following mathematical theorem. If the function f can be Fourier decomposed into the *positive frequencies only*, like in the equation above, then f cannot be zero for any finite range of $\tau = \tau_2 - \tau_1$, unless trivially it is zero everywhere. An immediate implication is that at the given x_1 the integral over \vec{k}_i in (10) cannot be zero for x_2 that is outside the light cone of x_1 . Hence the integral (10) necessarily involves the *nonzero* amplitudes that contain the space-like interval between x_2 and x_1 . If the interval is space-like, then the time order of the events is frame dependent. While in the rest frame one always has $\tau_2 > \tau_1$, i.e. all the virtual amplitudes describe propagation of particles forward in time, some of these amplitudes are seen in the moving reference frame as virtual particle that propagates backward in time, i.e. as antiparticle moving forward in time. This is a pure kinematics and cannot be circumvented.

It is important to realize, however, that this is a result of *interactions* of our particle with the other ones. Without interaction, i.e. when a particle propagates freely from the very beginning to the very end, all the intervals are time-like, i.e. the time order of the events is frame-independent. The free particle is seen as a particle in all frames. That is the Lorentz transformation of the system of particles without interaction (2). However once we boost the wave function of the system *with* interactions, where the individual 4-momenta of particles *are not defined* (such a wave function is a very complicated superposition of the wavefunctions of the type (3)) and which necessarily includes the space-like intervals for every *virtual* particle, these kinematical boosts include transformations of what we consider as virtual particles in the rest frame, to what which should be considered as *virtual* antiparticles in moving reference frame, i.e. it transforms the virtual amplitudes for positive energy particles with $\tau_2 > \tau_1$ to amplitudes with $t_2 < t_1$ in the moving frame. Once it is done, the wave function in the moving frame contains an admixture of *virtual* antiparticles, that propagate forward in time, though it is not seen explicitly in the time independent operator formalism [3]. Schematically it is shown in Fig. 1. What we observe as interactions between particles in the rest frame, Fig. 1a,

corresponds to creation from the vacuum (and annihilation) of the antiparticles by the *interaction* in the moving frame, Fig. 1b. The magnitude of those virtual amplitudes (with the space-like intervals) is completely specified by the interaction between particles.

The argument does not change if one assumes that at the point x_1 the particle of the system is coupled to the external field (e.g. electromagnetic one), propagates freely to the point x_2 and interacts there with other particles. Since the matrix element of the electromagnetic current, that is manifestly covariant [3], involves the system that either in initial, or in final states (or in both initial and final states) is not at rest, it automatically includes the “pair” currents, depicted in Fig. 2, in addition to the standard 1-body currents that contain no virtual antiparticles. Specifically, keeping in mind the model of ref. [5], this implies that the numerical result of ref. [4] does include the “pair” exchange currents associated with the confining interaction between quarks as well as with the Goldstone boson exchange.

We have used for simplicity only the amplitude that is quadratic in the interaction. It is obvious, however, that the same effect is present in all higher order amplitudes that contribute in the nonperturbative calculation.

That the “pair” exchange currents are very important is known from nuclear physics for a long time [12, 13]. In that case, i.e. when the nonrelativistic wave function is used and no Lorentz boosts are applied to wave function, these “pair” currents are introduced explicitly using the nonrelativistic v/c expansion of the corresponding Feynman diagram [14] so that the effect of these “pair” currents is absorbed into two- and three-body (etc.) operators that act in the Hilbert space of nonrelativistic wave function. These two- and many-body operators are used in addition to the nonrelativistic one-body current operator. The nonrelativistic one-body current represents the nonrelativistic impulse approximation. In the nonrelativistic impulse approximation only one particle (that is struck by the external field) is involved and other particles of the system are spectators. This is specified by the momentum delta-functions for the spectator particles that guarantee for this particles equality of their initial and final momenta $\vec{k}'_i = \vec{k}_i$ *both in the rest and in any moving frame* as well as by the fact that the spins of these spectator particles *are not affected*. The momentum transfer from the struck particle to the spectator one is provided only via wave function of the system that contains only particles and is exactly the same in the rest and moving frames. The two- and many-body “pair” current operators affect both the spins and momenta of two (many) particles and the momentum transfer from one particle to another one is provided by the operator, but not by the nonrelativistic wave function.

The point form relativistic impulse approximation [3] involves one struck particle and all other ones to be spectators only in that reference frame (e.g. the Breit one) where the matrix element is calculated. The wave function of the system in the initial and final states do not coincide with that one in the rest frame. The former wave functions

necessarily contain effects of creation of pairs from the vacuum by interaction between particles. These effects are implicitly introduced by boosting the wave function from the system rest frame, though boosts by itself are pure kinematic. It is important that this effect is completely specified by the interaction between particles that is used in the given model for the interaction in the system rest frame. The boosting affects spins of the “spectator” particles (which is provided by the corresponding Wigner rotations) as well as their momentum distributions. The latter is explicitly seen from the fact that instead of the condition $\vec{k}'_i = \vec{k}_i$, which is valid in the nonrelativistic impulse approximation, one actually has $\vec{k}'_i = B^{-1}(v_{final})B(v_{initial})\vec{k}_i$. Stated differently, boosting of the wave function from the system rest frame to the moving one, provides a set of two- and many-body operators that act on the space of the *rest frame wave functions*. These *include* effects of “pair” exchange currents introduced in the context of nonrelativistic nuclear physics. Contrary to the nonrelativistic scheme, however, the v/c expansion is not used and the “pair” currents are taken into account to all orders in interaction. Returning then to the calculation of nucleon electromagnetic formfactors in ref. [4] one concludes that the “pair” exchange currents that are associated with the Goldstone boson exchange between the constituent quarks and confining interaction are automatically included.

I am greateful to S. Boffi, W. H. Klink and D.O. Riska for a quick response and to A. Gal and the Nuclear Theory Group of the Racah Institute for hospitality and support during my visit to Jerusalem where the present work has been done.

References

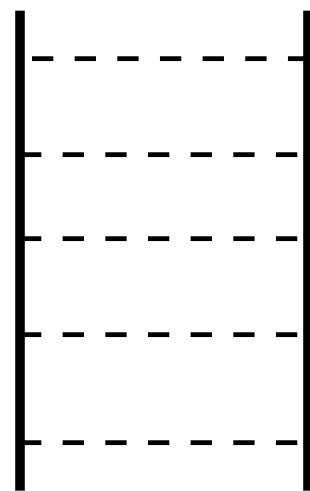
- [1] P. M. A. Dirac, Rev. Mod. Phys., **21** (1949) 392
- [2] B. D. Keister and W. N. Polyzou, Adv. Nucl. Phys., edited by J. W. Negele and E. W. Vogt (Plenum, NY), **20** (1991) 225
- [3] W. H. Klink, Phys. Rev. **C58** (1998) 3587
- [4] R. F. Wagenbrunn et al, Phys. Lett. B, submitted, nucl-th/0010048
- [5] L. Ya. Glozman, W. Plessas, K. Varga, and R. Wagenbrunn, Phys. Rev. **D58** (1998) 094030
- [6] L. Ya. Glozman and D. O. Riska, Phys. Rep. **268** (1996) 263.
- [7] L. Ya. Glozman and K. Varga, Phys. Rev. **D61** (2000) 074008
- [8] L. Ya. Glozman, Nucl. Phys. **A663** (2000) 103
- [9] B. Bakamjian and L. H. Thomas, Phys. Rev. **92** (1953) 1300

- [10] R. Feynman and S. Weinberg, Elementary particles and the laws of physics (The 1986 Dirac memorial lectures), 110 pp., Cambridge University press, 1987
- [11] W. H. Klink, Phys. Rev. **C58** (1998) 3617
- [12] D.O. Riska, in: Mesons in Nuclei, vol. 2, eds. M. Rho and D. H. Wilkinson (North-Holland, Amsterdam, 1979), p. 756
- [13] J.-F. Mathiot, Phys. Rep. **173** (1989) 63
- [14] M. Chemtob and M. Rho, Nucl. Phys. **A163** (1971) 1

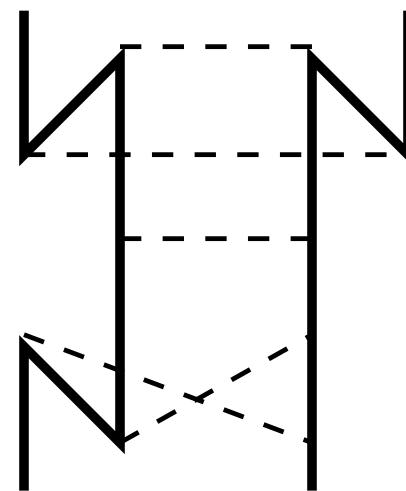
FIGURE CAPTIONS

Fig. 1 (a) Time evolution of particles as seen in the system rest frame; (b) the same as seen in the moving frame. The dashed line represent interactions.

Fig. 2 “Pair” current contributions to the electromagnetic observables, that involve creation of particle-antiparticle pairs by the interaction between particles in the system and by the electromagnetic field.



a



b

